3 The transformation of values into prices of production: a different reading of Marx's text

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The signs seemed the same but the words were different
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3.1 INTRODUCTION

Throughout the twentieth century, the transformation problem has been interpreted by Marx's critics as well as his defenders almost exclusively on the basis of the Ricardian approach established in 1906-7 by Bortkiewicz (1952, 1984).[1] The principal feature of this view is the methodological dualism established in the determination of values and prices. According to Bortkiewicz: 'the procedure employed by Marx for the transformation of values into prices is erroneous, since it fails to keep separate rigorously enough the two principles of value and price calculation.'[2] This separation involves the idea that 'prices' are dependent variables that must be 'derived' from 'values'. This approach – which will be called 'dualistic' here – ensues from a misunderstanding of Marx's dialectical analysis where the twofold nature of economic categories is always conceived of as a unity of opposites. Dualism misses such unity and replaces it with a cause and effect relationship wherein the poles of the categories are exclusively presented as separate realms.[3] In the case of the price-value relationship, the dualistic approach was first clearly proposed – though set out inversely – by Tugan Baranowsky (1905). Based on Tugan's vision, Bortkiewicz obtains his own well known result:

> It would not be permissible to equate total price with total value whilst simultaneously equating total profit with total surplus value. (Bortkiewicz 1952:12)

Once Sweezy (1970) gave the stamp of academic authority to this approach in 1942, it was developed by Winternitz (1948) and Seton (1957). In the 70s the debate intensified significantly because of Samuelson's article (1971) and Steedman's Sraffian reading (1977). These writers vigorously raised some of the issues established at the beginning of the century. The debate then involved many authors, such as Morishima (1973), Baumol (1974), Yaffé (1974), Gerstein

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(1976) and Shaikh (1977). Soon after, Duménil (1980), Foley (1982) and Lipietz (1982) proposed the so-called New Solution.[4] These authors essentially maintain the methodological dualism of the orthodox vision but consider that the transformation should be solved by only taking into account 'the net product'.

However, there is another group of authors that has begun to re-examine the methodological terms by which the debate has become crystallized. With different frameworks, the contributions by Wolff, Roberts and Callari (1982, 1984a), Carchedi (1984), Roberts (1987), Kliman and McGlone (1988), Giussani (1991), Freeman (1993a), and other writers have begun to break down the dualistic approach to the transformation. This point of view, however, is fully overwhelmed by the orthodox vision whose influence is almost absolute.[5]

One of the strengths of the dualistic approach is that a superficial reading of the draft left by Marx might mislead the interpreter towards finding what is *apparently* textual support of the traditional interpretation. One supporting element is that, in the first numerical example presented by Marx in order to illustrate his procedure in Chapter 9, *Capital* III, the figures corresponding to the cost price elements remain unchanged after the transformation is conceptually accomplished. The second supporting factor encompasses a group of passages where Marx discusses whether the deviation of the prices of inputs from their values affects his conclusions.

On this basis, Bortkiewicz established the two *leitmotifs* of the debate. First, Marx had left the transformation conceptually unfinished. Second, Marx had been conscious of this flaw but considered it unimportant; in support of this assertion the above mentioned texts began being cited. Actually, these points were first made (and one of these texts quoted) by Komorzynsky (1897), who is a veritable 'missing link' between Böhm-Bawerk and Bortkiewicz.

The purpose of this chapter is to read systematically the procedure developed by Marx in Chapter 9 of *Capital* III in the light of the above mentioned methodological reworking of the transformation problem. A thorough reading of the text will show that the alleged evidence has a completely different meaning in relation to the interpretation put forth by Bortkiewicz. In the first section the thesis maintained by Marx will be followed; in the second section an interpretation of the methodological meaning of the transformation in Marx's presentation is advanced; in the third section the main critiques of Marx's procedure will be traced. In the fourth and fifth sections a method of carrying out the transformation will be presented, which, in contrast with conventional methods, tries to emphasize the Marxian conception of the relationship between value and its form. The conceptual transition achieved by Marx in the second and third tables of Chapter 9, *Capital* III, is the initial step of this procedure whose complete development will corroborate the soundness of Marx's conclusions. The last section will carefully examine the passages where Marx tests the validity of his results. To clarify his reasoning, we will follow an observation made by Marx himself in one of these texts where he refers the reader to a method used in

Capital I and II. Thanks to this remark, completely neglected by subsequent literature, his texts can be read with a new and different meaning. Moreover, it will be shown that the passage quoted by Duménil (1980) does not support his interpretation of the transformation.

3.2 VALUE AND PRICE IN MARX

The terms of the transformation problem were presented by Marx in *Capital* I:

> The masses of value and of surplus-value produced by different capitals – the value of labour-power being given and its degree of exploitation being equal – vary directly as the amounts of the variable components of these capitals, i.e. the parts which have been turned into living labour-power. This law clearly contradicts all experience based on immediate appearances. Everyone knows that a cotton spinner, who, if we consider the percentage over the whole of his applied capital, employs much constant capital and little variable capital, does not, on account of this, pocket less profit or surplus-value than a baker, who sets in motion relatively much variable capital and little constant capital. For the solution of this apparent contradiction, many intermediate terms are still needed .[6]

When commodities are exchanged at *prices* corresponding to their values, the surplus value *appropriated* by the different spheres – called *profit* by Marx – equals the surplus value *produced* by them. However, their exchange tends to be accomplished in proportions determined by the amounts of capital advanced in production, that is, according to the production prices rather than the values of commodities. Actually, they are exchanged at their market prices and production prices are only tendentially imposed in competition between individual capitals.

Grasping the contradiction which Marx discusses in the above passage requires comprehending the concept of value and, particularly, the relation between value and its form. It is usual to define erroneously value as 'labour', that is, to reduce value to its substance.[7] Actually, value is a complex concept: value is the unity of abstract labour (its substance) and money (its form)[8] and, thus, it has an immanent or intrinsic measure (socially necessary labour time) and an extrinsic measure (exchange value or price).

In capitalist society, labour is realised as social labour under the form of money. Marx always refers to value as a quantity of money because

> [m]oney as a measure of value is the necessary form of appearance of the measure of value which is immanent in commodities, namely labour time. (Marx 1976a:188)

Measuring value in labour time units shows a misunderstanding of the 'internal, necessary connection between the *form* of value, the *substance* of value and the *magnitude* of value.'[9] Therefore, the value of a commodity can only be expressed through a given quantity of another commodity's use value; when the latter is the money commodity, this expression – that is, the amount of the use value of the money commodity as measured in units determined by the standard of prices (for example an ounce of gold) – is called *price*. Yet, the value objectified in the quantity of use value which constitutes the commodity's price

may diverge from the value embodied in the commodity itself. Thus, the price of the commodity

> may express both the magnitude of value of the commodity and the greater or lesser quantity of money for which it can be sold under the given circumstances. The possibility, therefore, of a quantitative incongruity between price and magnitude of value, i.e. the possibility that the price may diverge from the magnitude of value, is inherent in the price-form itself. (Marx (1976a:196)

Through his critique of Ricardo, Marx understood that the incongruity between price and value is not limited to occasional divergences but, rather, constitutes one of the key features of capitalist competition. By elucidating the rationale of these divergences, that is, explaining the contradiction between price and value, the analysis of the relation between value and its form is brought to a more concrete level and, at the same time, the presentation of how the law of value acts through competition is made more complex.

Marx deals with this issue in many passages[10] and presents its solution in tabular form on five occasions, two of which are in Chapter 9, *Capital* III.[11] Since Komorzynsky and Bortkiewicz, the literature has been concerned with the first example of this chapter, where Marx presents a numerical example with five spheres of production developed in three consecutive tables. In the first table, each sphere advances a global capital of 100, which is completely consumed in production so that the value produced by each sphere can be broken down into capital advanced and surplus value. In the second table, all the spheres advance the same capital of 100, but a fraction of it is not consumed; in this case the value produced by each sphere is broken down into the consumed fraction of the capital advanced (cost price) and the surplus value.

In both tables, Marx assumes that the rate of surplus value is the same in all spheres (100 per cent) but their organic composition is different. This implies that, with the same advanced capital of 100, each sphere exploits a different quantity of living labour and thus produces a different amount of surplus value. Marx's second table[12] is laid out as in table 3.1below:[13]

	Constant Capital	Constant Capital Used Up	Variable Capital	Cost Price	Surplus Value	Value	Rate of Profit (%)
#1	80	50	20	70	20	90	20
#2	70	51	30	81	30	111	30
#3	60	51	40	91	40	131	40
#4	85	40	15	55	15	70	15
#5	95	10	5	15	5	20	5
Σ	**390**	**202**	**110**	**312**	**110**	**422**	**22%**

Table 3.1

It is important to note that Marx does not say what is the unit of measure of these magnitudes. This is one reason why many authors, once the transformation debate began, measure value in labour units and not money.[14] Actually, the unit of measure can only be money because, as Marx states on many occasions,

money, as a measure of value is the necessary form of appearance of the measure of value which is immanent in commodities, namely labour time.[15]

This is confirmed, for instance, in passages of *Grundrisse, Theories of Surplus Value* and Marx's letter to Engels dated August 2, 1862, where he also explains the transformation of values into prices of production.[16] In these examples all value magnitudes are expressed in money (£). The omission of units of measurement in the corresponding text of *Capital* III only shows the provisional and unfinished character of the draft published by Engels.

What does the sixth column of Table 3.1 represent?

[The] *money prices* at which [the] commodities would exchange if they were exchanged according to their *values.*'[17]

Therefore, this column simultaneously depicts the *value* and the *price* of the product of each sphere, while the fifth column simultaneously represents the surplus value *produced* by each sphere and the *profit* or *surplus value* they have appropriated.[18] Given the different organic compositions of the five spheres, this assumption would imply that the profit rate of the various spheres – calculated as the ratio between their profit and their invested capital and shown in the seventh column – diverge from one another. However,

in actual fact, ignoring inessential, accidental circumstances that cancel each other out, no such variation in the average rate of profit exists between different branches of industry, and it could not exist without abolishing the entire system of capitalist production. (Marx 1981: 252)

Instead of calculating the *value-price vector* presented in column 6 of Table 3.1, it is also possible to calculate another *price vector* which distributes the surplus value produced between all spheres in proportion to the invested capital; that is, a vector of prices of production. To do this, it is necessary to calculate the general rate of profit (π) as the ratio between the mass of surplus value produced (SV) by the society, and the *total* capital advanced (constant C plus variable V capital):

$$\pi = \frac{SV}{C + V}$$

In his third table (table 3.2) Marx calculates this rate of profit and the prices of production corresponding to each sphere:

	Constant Capital	Constant Capital Used Up	Variable Capital	Cost Price	Surplus Value	Value	Profit	Price of Production	Diverg-ence
#1	80	50	20	70	20	90	22	92	+2
#2	70	51	30	81	30	111	22	103	-8
#3	60	51	40	91	40	131	22	113	-18
#4	85	40	15	55	15	70	22	77	+7
#5	95	10	5	15	5	20	22	37	+17
Σ	390	202	110	312	110	422	22	422	0

Table 3.2

The price of production of each branch is broken down into cost price and average profit which is calculated as the proportion π of the total capital invested (not only the consumed capital or cost price). The migration of capital across the different spheres of the economy in search of a higher profit rate tends to equalise the sectoral rates of profit. As a result, prices on average tend to correspond to production prices, determined by the amount of *capital* necessary to produce the commodities, rather than to their values, determined by the amount of *labour* necessary to produce them.[19]

The difference between value and price implies that the surplus value *appropriated* through exchange among the various branches – profit – no longer coincides with the surplus value *produced* by each of them. Those branches that exploit relatively more labour sell their commodities at a price lower than their value and thus pocket a profit lower than the surplus value they have extracted; the reverse occurs in the spheres that mobilize relatively less living labour. However, taking all the spheres together,

> the divergences of price from value … cancel each other out when surplus value is distributed evenly … To the same extent that one section of commodities is sold above its value, another is sold below it. (Marx 1981:257)

Therefore,

> if a commodity is sold above or below its value, there takes place merely a change in the distribution of surplus value between different capitalists. (Marx 1991:75)

If commodities are sold at their prices of production, how does value 'determine' these prices?

> It is clear that, however much the [price of production] of an individual commodity may diverge from its value, it is determined by the *value* of the total product of the social capital.[20]

The fact that value constitutes a 'determinant' means that it is a quantitative *limit* established by total product; prices of production simply represent a redistribution of this produced quantity of value. Hence, *determination* is not a cause and effect relationship.

These are the main features of Marx's solution to the apparent contradiction between value and its form which he points out in the *Capital* I passage quoted at the beginning of this section: *value* results from the objectification of socially necessary labour; *price* ensues from the distribution of the surplus value among the various branches of production. As is well known, Marx's solution implies that the sum of values equals the sum of prices of production and, at the same time, the sum of surplus value equals that of profits.[21] This conclusion was later to be criticized by Tugan Baranowsky, Bortkiewicz and many others.

Yet, before considering these critiques, it is convenient to discuss the methodological meaning of the transformation and, specifically, the transition accomplished between the second and the third tables of Chapter 9, *Capital* III.

3.3 THE METHODOLOGICAL MEANING OF THE TRANSFORMATION

The dualistic approach to the transformation considers that, before *Capital* III, Marx has exclusively dealt with the 'value calculation' as completely separate from any price expression. This interpretation involves the belief that values are a 'system' separate from prices and even expressed in a different unit of measure (labour time). According to Dobb, the transformation problem

> is essentially whether or not the prices of production … are *deducible* from … value[s], as determined by quantities of embodied labour. (Dobb 1955:273, emphasis added)

Afterwards, it is commonly asserted that

> a set of … equations can be used to express the value of each output as the amount of labour used directly [and] indirectly … these values are entirely determined by technological relationships and … entirely independent of pricing. (Baumol 1974:56)

It has been generally supposed that the relationship between Marx's tables is one of 'causation', where the third (the 'price system', belonging to 'circulation') is 'derived' from the second (the 'value system', 'entirely independent of pricing').

The defenders of this approach attempt to be faithful to Ricardo's and Marx's critiques of Smith's circular adding up concept of value.[22] Nevertheless, when Marx says that value 'comes first' or 'is prior to' the price of production, he means that value is the form in which social labour is objectified and price is the form in which it is appropriated; this does not mean, however, that value is a causal factor determined before prices.[23]

One complementary version of the dualistic interpretation is based on Rosdolsky (1977). According to this author, in *Capital* I and II, Marx considers 'capital in general' and completely abstracts from the multiplicity of capitals, competition and prices, elements that are allegedly taken into account in *Capital* III. However, neither Rosdolsky nor his followers (Moseley 1993a) have been successful in demonstrating how the operation of the law of value can be accomplished outside of competition, that is, outside the concrete process where the prices are formed. Competition is an inherent element not only of the concept of capital but also of the concept of value itself.[24]

If competition and prices are regarded, what is Marx's abstraction before setting out *Capital* III? Marx specifically abstracts from the fact that 'the existence [of] a general rate of profit … prima facie contradicts the determination of value by labour time',[25] that is the contradiction between values and prices as presented in the passage of *Capital* I, Chapter 11, cited at the beginning of the first part of this chapter. In this text, Marx summarizes his critique of Ricardo and states that 'many intermediate terms are still needed' to grapple with this opposition between essence and appearance. Only when

> the transformation of … labour-power into wages [and] the transformation of surplus-value into profit … ha[ve] been explained.[26]

can this contradiction be properly resolved. Beforehand, it is necessary to abstract from this issue and, correlatively, to assume that 'prices = values'. Yet, this assumption by no means signifies that value is a substance lacking form; that is, that commodities have no price.

Therefore, from the beginning Marx has taken competition into account, but under conditions that imply that the vector of values coincides with that of prices ('values = prices'); that is, competition is considered as a *formal* but *existing* process.

Having developed the required categories, Marx works out his second table (Table 3.1) – where surplus value has been transformed into profit[27] – maintaining the preceding assumption that 'prices = values' and considering the heterogeneity of capital compositions.[28] This table, wherein commodities have value and price, does not represent a 'system of values', 'entirely independent of pricing', as the dualistic approach claims. When the heterogeneity of capital compositions is taken into account, the result is the emergence of different rates of profit, an outcome that contradicts the immanent tendency towards a general rate of profit. These are the terms of the contradiction – abstracted from until *Capital* III – that Marx seeks to resolve; to do this, he accepts that *prices ≠ values* and correspondingly introduces the category of price of production. Therefore, from now onwards *prices = production prices*. As seen, this is how the third table (Table 3.2) is worked out. Marx resolves the contradiction by showing that there is a permanent divergence between values and prices which produces a transfer of surplus value among the spheres. The consideration of competition, previously taken into account *only formally*, becomes a *real* element of the presentation.

Yet this step in the construction of the concrete totality of the capitalist reproduction means that, in contrast to *Capital* I and II, the law of value is *negated* as the norm of exchange between *individual commodities*, and that value and its form no longer coincide directly. In particular, a divergence between the *intrinsic* measure of value (labour time) and its *extrinsic* measure (value in exchange) arises. In his third table Marx shows, however, that this contradiction of the law of value is produced *at the level of individual capitals* but is superseded *at the level of the totality* of capitalist circulation. This is the meaning of the global annulment of the divergences between prices and values: it means that the individual differences between value and price – that is the negation of the law of value as the norm of individual exchange – are the concrete form through which *value* becomes the expression of social labour.[29] Therefore, it is clear that 'production prices [are] mere transformed forms of value'[30] and that, considering the totality, they are only fractions of value – specifically, the forms under which value has been appropriated.

There is another sense according to which the principle of construction of the second table is preserved in the third table. In his second table Marx presents commodity *values* as formed by the *price* paid for the elements of cost price and by the surplus value. This calculation principle has been kept in the third table

but here the *prices* of the elements of cost price are equal to their respective *prices of production*, whereas in the second table they are equal to their *values*.

The transformation of values into prices of production is, therefore, a dialectical transition in the presentation of the operation of the law of value and of the relation between value and its form: the simple form of the law of value (values = prices) is negated *at individual level* by the equalisation of the rate of profit (values ≠ prices of production) but this is only the manner in which it is imposed on the *totality* (sum of values = sum of prices). In this way, the law of value becomes a complex category and the relation between value and its form *actually* presents the feature of 'quantitative incongruity' which is inherent in its development as price form. Thus, Marx's equalities are *unities of opposites* which express the two contradictory aspects of one process, more specifically, the contradictory unity of production and distribution.[31] Marx elucidates the rationale of this contradiction in a series of mediations, not completely developed in the first section of *Capital* I.

This transition is expressed in the relation between the two main tables of Chapter 9, *Capital* III: the third table *negates* and simultaneously *preserves* the criterion of construction of the second table; that is, the third table *contradictorily contains* the second. Tugan Baranowsky and Bortkiewicz read this transition erroneously, believing that each table represents a 'world' completely apart from the other.[32] In particular, they think the third table is 'derived' mechanically from the second, without perceiving that the latter is integrated in the former.

3.4 TUGAN BARANOWSKY'S AND BORTKIEWICZ'S CRITIQUES OF MARX'S METHOD

During the early twentieth century, the tables drafted by Marx were the object of two reworkings which have framed the 'modern' transformation debate. The first was Tugan Baranowsky's.[33] It constitutes the basis of the second, carried out by Bortkiewicz[34] a few years later. This has since been at the heart of the whole debate.

Bortkiewicz criticizes the procedure Marx followed in drafting his solution. He slightly modifies Marx's second table (Table 3.1) to convert it into a simple reproduction schema: constant capital consumed by sphere 2 is 50 in lieu of 51, and in sphere 3, 52 instead of 51. Moreover, he assumes that spheres 3 and 4 produce means of production; 1 and 5 means of subsistence for the workers; and sphere 2 luxury goods. Marx's table, as modified by Bortkiewicz, becomes Table C. On this basis, Bortkiewicz reworks Marx's third table, where production prices are calculated. This is shown in Table 3.4.[35] Bortkiewicz's critique of Marx's presentation is clearly shown in this table.[36] Here, the general rate of profit is calculated, as in Table 3.3, as the ratio between total surplus value (110) and the total capital invested (500); this rate is then used to determine prices of production.

	Constant Capital	Constant Capital Used Up	Variable Capital	Cost Price	Surplus Value	Value	Rate of Profit (%)
I	**145**	**92**	**55**	**147**	**55**	**202**	
#3	60	52	40	92	40	132	40.0
#4	85	40	15	55	15	70	15.0
II	**175**	**60**	**25**	**85**	**25**	**110**	
#1	80	50	20	70	20	90	20.0
#5	95	10	5	15	5	20	5.0
III	**70**	**50**	**30**	**80**	**30**	**110**	
#2	70	50	30	80	30	110	30.0
Σ	**390**	**202**	**110**	**312**	**110**	**422**	**22.0**

Table 3.3

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	Constant Capital	Constant Capital Used Up	Variable Capital	Cost Price	Surplus Value	Value	Profit	Price of Production	Diverg -ence	Quan -tity	PP/Q
I	**145**	**92**	**55**	**147**	**55**	**202**	**44**	**191**	**-11**		
#3	60	52	40	92	40	132	22	114	-18	33	3.45
#4	85	40	15	55	15	70	22	77	+7	35	2.20
II	**175**	**60**	**25**	**85**	**25**	**110**	**44**	**129**	**+19**		
#1	80	50	20	70	20	90	22	92	+2	30	3.07
#5	95	10	5	15	5	20	22	37	+17	5	7.40
III	**70**	**50**	**30**	**80**	**30**	**110**	**22**	**102**	**-8**		
#2	70	50	30	80	30	110	22	102	-8	55	1.85
Σ	**390**	**202**	**110**	**312**	**110**	**422**	**110**	**422**	**0**		

Table 3.4

Bortkiewicz finds that, in the calculation of these prices, Marx kept the figures corresponding to the *value* of the elements of the cost price unaltered, instead of transforming them into *prices of production*; that is Marx

> made the mistake of carrying over certain magnitudes without alteration from the table of values into that of prices. In transforming values into prices, it is inadmissible to exclude from the recalculation the constant and variable capital invested in the various spheres of production. (Bortkiewicz 1952:9)

Actually, this proposition was first advanced by J. V. Komorzynsky. He asserts:

> Marx has disregarded the mutual dependence of the prices of the various products and the same omission is found in many passages where he presents the 'price of production' as 'cost-price' including profit but, at the same time, he defines 'cost-price' as the 'value' of the consumed constant and variable capital. [For example, Marx asserts that] 'prices of production … are equal to their cost elements (the value of the constant and variable capital consumed) plus a profit determined by the general rate of profit'.[37]

It is clear that, in his reading, Komorzynsky confuses the value of the *constant* and *variable capital* – that is, a sum of money devoted to purchase the inputs, which is generally equal to their prices of production – with the value of the *means of production* and the value of the *wage goods*. Besides this, Komorzynsky asserts that Marx was conscious of his own neglect because he 'was fully familiar with the mutual interaction of product prices'. To support this

statement, Komorzynsky quotes a passage of *Capital* III, Chapter 9.[38] In this form, this author introduced a reading error which has been accepted uncritically in all the subsequent literature. Summing up these propositions, Bortkiewicz asserts that Komorzynsky

> shows how Marx has not consequently carried out the conversion of values into prices [and] … finds that, in *Capital*, the expressions of value and price are confused one with another and, in this point, his critique is an important complement to Böhm-Bawerk's.[39]

He concludes that Marx's solution

> cannot be accepted because it excludes the constant and variable capital from the transformation process, whereas the principle of the equal profit rate, when it takes the place of the law of value in Marx's sense, must involve these elements. (op cit p201)

Tugan Baranowsky also points out a problem he finds in Marx's procedure. The rate of profit in Marx's third table (Table 3.2) is calculated as the ratio between the social surplus value and the *value* of the elements of the invested capital, that is the value of means of production plus the value of wage goods. But Tugan observes that, when capital is invested, the actual rate of profit taken into account by capitalists is not determined by this ratio, but rather by the ratio between the surplus value produced and the *price* of the elements of the invested capital:

> We can see, thus, that the general or social rate of profit differ, depending on whether its calculation is carried out with commodities' money prices or labour values. Yet, which of these two rates has relevance? Obviously the rate calculated in accordance with money prices because profit arises actually on the basis of money prices.[40]

A proper calculation would lead Marx into a vicious circle: Marx needs the rate of profit to calculate the correct production prices but he also needs the latter to obtain the former.

These two problems lead Tugan Baranowsky and Bortkiewicz to assert that the transformation procedure devised by Marx was left theoretically unfinished. In order to complete it Bortkiewicz, using Tugan Baranowsky's example, applies a method that – he thinks – rectifies Marx's mistake. As a consequence of this rectification, Bortkiewicz asserts that Marx's double equality is not valid.[41] Although his work is less algebraically elaborated, Tugan Baranowsky reaches a similar conclusion.[42]

It can be presumed that the transformation is incomplete; that is, that Marx's numerical tables are unfinished. In his draft, he illustrated the general lines of his method of calculation (for example the definition of the rate of profit) but he did not construct a finished numerical example. This is probably why, in his three tables, the figures corresponding to the cost price elements and the rate of profit are the same. To 'complete the transformation' merely means to carry out a numerical calculation which does not affect Marx's theoretical framework.

In his attempt to 'correct' the transformation, Bortkiewicz misinterprets Marx's conception of the relation between value and its form. In particular, he misunderstands the theoretical meaning of the assumption that values = prices

that – it can be assumed – Marx maintains in the first two tables of Chapter 9, *Capital* III. Bortkiewicz thinks that in these two tables Marx only determines the commodity's values, while in the third he only determines prices; the first two tables then represent a world of values without prices, the latter is a world of prices without values. Hence it is not by chance that in reworking Marx's third table Bortkiewicz suppresses the column corresponding to values: according to him values were determined, once and for all, in the previous table.[43]

Marx, on the contrary, in each of his tables simultaneously determines values and prices. As already stated, the assumption 'values = prices' in the first two tables means that the column of values represents, *at the same time*, the *value* and the *price* of commodities, that is that commodities are exchanged in proportions that allow their producers to appropriate all surplus value extracted from the workers. Therefore, this assumption does not imply a calculation without prices. When Marx passes from the second to the third table, he calculates the prices that allow the capitalists to appropriate a uniform portion of the total surplus value; nevertheless, in the third table, *he calculates values as well as prices*. The fact that the commodities are now exchanged with prices differing from their values

> is not a defect, but, on the contrary, it makes this form the adequate one for a mode of production whose laws can only assert themselves as blindly operating averages between constant irregularities. (Marx 1976a:196)

How can it be shown that Marx's calculation in each of his tables is simultaneously a value and price determination, in contrast with Tugan Baranowsky's and Bortkiewicz's? According to these authors, the value of a commodity is formed by the sum of the *value* of means of production, the value of wage goods and the *value* of commodities appropriated by capitalists (luxury goods in simple reproduction). This concept of the magnitude of value coincides with Marx's before *Capital* III. However, when prices no longer correspond to values, Marx states that the *value* of the commodities is broken down into constant capital, variable capital and surplus value. Constant capital is a given amount of money that the capitalist allots to the replacement of means of production and, thus, does not necessarily match their *value* – as Tugan Baranowsky and Bortkiewicz believe – but their *price*. Variable capital is the amount of money allotted to wages, which is used by workers to buy their means of subsistence, and corresponds to the *price* of the wage goods and not to their value. Finally, surplus value is the difference between the new value produced by living labour and the wages. Conceived in this form, value can only be understood as the result of a process where value is determined at the same time as price. Value is not – as Tugan Baranowsky and Bortkiewicz think – a magnitude given separately from prices and the circulation of commodities. Value and price are dialectically linked and form the contradictory unity of value and its form.[44] The dualistic method used by Bortkiewicz, supposedly to correct the transformation, is based on an understanding of value different from Marx's, leading to incorrect conclusions.

3.5   A MATERIAL REPRODUCTION SCHEMA

An alternative method to complete the transformation procedure will now be presented, which corroborates his conclusions. This method eliminates the dualistic vision of the price-value relation and follows Marx's presentation in the first example of Chapter 9, *Capital* III. Although the simple reproduction schema devised by Bortkiewicz will be used (Table 3.3), the presentation will show that Marx's solution is consistent.

The simple reproduction schema permits use values and values to be distinguished from one another. As Marx stresses in *Capital* II, there are, behind the exchange relations between the different departments, specific proportions in which the different use values must appear in order to allow the material reproduction of society.[45] The value relations between the various departments can only be established on the basis of the exchange of specific use values. When Marx drafted his *general outline* of the transformation, he did not need to make the material proportions underlying his tables clear; however, the calculation of prices and values here needed to complete his transformation procedure requires making these material relations explicit.

Table 3.3 assumes a given proportionality between the physical supply and demand of the different spheres. The total value of the consumed means of production is 202, 65.3 per cent of which is produced by sphere 3 and 34.7 per cent by sphere 4. This value of 202 is, at the same time, the total constant capital consumed by all spheres. Bortkiewicz is not explicit about how the constant capital of each sphere is proportionally formed by means of production from spheres 3 or 4. In order to simplify these calculations, without affecting his results, it can be assumed that, in all spheres, the different means of production are combined as inputs in the same proportion by which they are produced as outputs; a similar assumption can be made for the means of consumption. For example, sphere 3 consumes £52 of constant capital: 65.3 per cent corresponds to the value of means of production produced by 3 and 34.7 per cent by sphere 4; this implies that sphere 3 consumes £52×0.653 = £34 of sphere 3's global production and £52×0.347 = £18 of sphere 4's total production.

Nor is Bortkiewicz explicit about the amounts of use values behind the value reproduction schema. However, any physical output whatsoever can be assumed for each sphere. The ratio between the total value of each sphere and the given amount of use values is equal to the unit value of the commodity. For instance, it can be supposed that the five spheres produce 30, 55, 33, 35 and 5 units of their use values, so that the unit values are £3, £2, £4, £2 and £4, respectively. These physical outputs are subsequently allocated to the different spheres in the same proportions by which their values are distributed. For instance, sphere 3's advanced constant capital (£52 = £34 + £18) purchases £34/£4 = 8.5 units of means of production 3 and £18/£2 = 9 units of means of production 4.

	Constant Capital		Constant Capital Used Up		Variable Capital		Supply
	Sphere #3 (1)	Sphere #4 (2)	Sphere #3 (3)	Sphere #4 (4)	Sphere #1 (5)	Sphere #5 (6)	(7)
#3	9.8	10.4	8.5	9.0	10.9	1.8	33
#4	13.9	14.7	6.5	6.9	4.1	0.7	35
#1	13.1	13.9	8.2	8.7	5.5	0.9	30
#5	15.5	16.5	1.6	1.7	1.4	0.2	5
#2	11.4	12.1	8.2	8.7	8.2	1.4	55
Σ	63.7	67.6	33.0	35.0	30.0	5.0	

Table 3.5

Applying this procedure to all the elements of each sphere's constant and variable capital one can create a table (Table 3.5) which makes the material proportions underlying Bortkiewicz's reproduction schema explicit.[46]

Columns 1 and 2 represent the amount of means of production used as fixed capital by each sphere, and columns 3 through 6 show the quantities of the different commodities consumed in production. Since a balance between supply and demand has been assumed throughout, the total of the last four columns (the physical demand for each kind of commodity) equals column 7 (the physical supply of each use value). The ratio of the value produced by each sphere (column 6 of Table 3.3) and physical production of each sphere (the last column of Table 3.5) gives the vector of unit values: £3, £2, £4, £2, £4.

3.6   AN ALTERNATIVE PROCEDURE FOR COMPLETING MARX'S TRANSFORMATION

The method used by Marx to illustrate the transformation in the third table of *Capital* III, Chapter 9 (Table 3.2) can be interpreted as the first in a series of approximations of the final calculation of the prices of production and the rate of profit as well as the values themselves.[47]

To complete the procedure, Bortkiewicz's modified version of Marx's third table will be used. However, three columns have been added to this table:

Column 6   corresponds to values and was originally suppressed by Bortkiewicz.

Column 10   shows the physical production of each sphere and is equal to the last column of Table 3.5.

Column 11   is the first calculation of the vector of unit prices of production, calculated by dividing the production price of each sphere (column 8) by the physical quantities produced (column 10).

It can be conjectured that in Table 3.3 prices correspond to values and that, in Table 3.4, Marx uses this prevailing price vector to evaluate the inputs. To the resulting cost price, Marx adds the average profit and obtains an initial *provisional* price of production, shown in column 11 of Table 3.4. Obviously, this

is not the final outcome of the calculation since the same use value is being evaluated using two different *prices,* depending on whether it is an input or an output. Hence, it is necessary to complete the procedure through successive iterations. Table 3.6 presents the result of the next iteration.

To perform the second iteration, the amount of physical inputs and outputs, as presented in Table 3.5, remains unchanged and is evaluated using the new vector of production prices, obtained from Table 3.4 (column 11). The difference between Table 3.4's total production price (422) and the new total cost prices of Table 3.6 (320) is the new approximation to the total surplus value (102). The parts of this surplus value produced by the different spheres are determined to be a homogeneous proportion of their variable capital. The value produced by each branch is formed by its produced surplus value and its cost price, constituted by the *price* of the means of production and the *price* of the wage goods (not by their *values*, as imagined by Bortkiewicz).

	(1) Con-stant Capital	(2) Con-stant Capital Used	(3) Var-iable Cap-ital	(4) Cost Price	(5) Sur-plus Value	(6) Value	(7) Profit	(8) Price of Produc--tion	(9) Diver-gence	(10) Quan-tity	(11) PP/Q
I	137.1	87.0	64.5	151.5	51.0	202.5	41.3	192.8	-9.7		
#3	56.7	49.2	46.9	96.1	37.1	133.2	21.2	117.3	-15.9	33	3.56
#4	80.4	37.8	17.6	55.4	13.9	69.3	20.1	75.5	6.2	35	2.16
II	165.5	56.7	29.3	86.1	23.2	109.2	39.9	126.0	16.7		
#1	75.6	47.3	23.5	70.7	18.5	89.3	20.3	91.0	1.8	30	3.03
#5	89.8	9.5	5.9	15.3	4.6	20.0	19.6	34.9	15.0	5	6.99
III	66.2	47.3	35.2	82.5	27.8	110.3	20.8	103.2	-7.0		
#2	66.2	47.3	35.2	82.5	27.8	110.3	20.8	103.2	-7.0	55	1.88
Σ	368.8	191.0	129.0	320.0	102.0	422.0	102.0	422.0	0.0		

Table 3.6

The ratio between the total surplus value and the capital invested by all spheres (368.8 + 129.0) gives a new estimation of the general rate of profit which is, in this second iteration, 0.205. With this figure, the average profit of each branch is calculated and then added to the cost price, leading to a new total price of production. These prices do not correspond to those calculated in the previous iteration and used to evaluate the elements of the advanced capital. Therefore, the calculation has still not been completed and it is necessary to repeat the procedure. However, after several iterations, the system converges to a point where the prices of production of the outputs corresponds to the prices of production of the inputs, which completes the example whose first stage Marx drafted in his third table. The final result of the iterations is Table 3.7:

	(1) Constant Capital	(2) Constant Capital Used Up	(3) Variable Capital	(4) Cost Price	(5) Surplus Value	(6) Value	(7) Profit	(8) Price of Production	(9) Divergence	(10) Quantity	(11) PP/Q
I	138.2	87.7	63.2	150.9	51.5	202.4	41.7	192.6	-9.9		
#3	57.2	49.6	46.0	95.5	37.5	133.0	21.3	116.9	-16.1	33	3.54
#4	81.0	38.1	17.2	55.4	14.1	69.4	20.3	75.7	6.3	35	2.16
II	166.8	57.2	28.7	85.9	23.4	109.3	40.5	126.4	17.0		
#1	76.3	47.7	23.0	70.6	18.7	89.4	20.5	91.2	1.8	30	3.04
#5	90.6	9.5	5.7	15.3	4.7	20.0	19.9	35.2	15.2	5	7.04
III	66.7	47.7	34.5	82.1	28.1	110.2	20.9	103.1	-7.2		
#2	66.7	47.7	34.5	82.1	28.1	110.2	20.9	103.1	-7.2	55	1.87
Σ	371.8	192.6	126.4	318.9	103.1	422.0	103.1	422.0	0.0		

Table 3.7

At this point, the reader might be tempted to compare the results of the last iteration with those of the first, as if the figures represented two historically different moments. That is to say, as if the first iteration corresponded to a situation of disequilibrium and the last to a situation when the price system had converged to equilibrium. The reader might also think – as Bortkiewicz does and Shibata even more clearly – that the *true values* of the commodities are those of the 'zero' iteration (Table 3.3) and that their *true prices of production* are those of the last iteration (Table 3.7). Such a reading is incorrect. The interpretation of the relationship between both tables has already been presented in the second section of this chapter. In the present numerical illustration, the magnitudes arising from the 'zero' iteration in Table 3.3, or from first iteration, in Table 3.4, are the intermediate results of *calculation process* since – assuming that there is no technical change – it is quite impossible for commodities to have two prices, one as outputs and another as inputs. More generally, it is impossible for the prices of commodities (as inputs or outputs) to correspond to their values. The magnitudes of Table 3.4 are only numerical approximations of the true *value and price* magnitudes (Table 3.7) whose calculation passes through either a series of iterations such as those illustrated or the solution of a system of simultaneous equations.[48]

Additionally, it is important to keep in mind that all the figures in the above tables are measured in money (£). The representation of these magnitudes in labour time requires the determination of the *monetary expression of labour,* a relation between the extrinsic and the intrinsic measures of value which will not be considered here.[49] Nonetheless, it is important to draw three important conclusions from the final result of the iterative procedure. First, there is a single general rate of profit in the system, defined – as Marx wanted – by the ratio between the mass of surplus value and the sum of the capitals invested in all spheres. In this case, it is

$$\pi = \frac{SV}{C + V} = \frac{103.1}{371.8 + 126.4} = 0.207$$

Given this result, it is clear that Tugan Baranowsky's observation that Marx's system has two alternative rates of profit – 'in value terms and in money terms' – is groundless.[50] Second, there is a rigorous verification of Marx's result, where the sum of the profits for all the different spheres of production must accordingly be equal to the sum of surplus-values, and the sum of prices of production for the total social product must be equal to the sum of its values.[51]

Third, the system represented in Table 3.7 is, *at the same time*, a value system and a price system. The price and value vectors, as well as the rate of profit and the rate of surplus value, are not determined independently of each other; they are, rather, results of the same process of determination, that is the competition through which capitalist reproduction evolves. This is in complete contrast with Bortkiewicz's interpretation of Marx's third table and, in general, with the orthodox reading of the transformation. According to Bortkiewicz, Marx thinks that 'the very existence of the rate of profit suspends the law of value,'[52] in a such way that the transformation is the passage from a system where there are only values to another where there are only prices.

3.7   A COMMENT ON MARX'S CONTROVERSIAL TEXTS ON THE TRANSFORMATION

Having established how the quantitative divergences between values and prices are formed, and having illustrated them using the numerical example above, Marx then investigates a series of circumstances which may affect the consistency of his drafted solution, in particular the double equality. His concern is expressed in a series of passages following his tables, which have been quoted repeatedly and generally out of context in the transformation debate. In this section three of these passages, probably those most frequently quoted, are examined.[53] In each passage, Marx's problem as well as his conclusion will be made explicit. They invariably confirm his solution of the transformation problem. To illustrate Marx's reasoning the economy represented in Table 3.7, where the transformation procedure has been completed, is used as an example.

First problem: is there double counting of profit when all commodities (including the inputs) are exchanged at their production prices?[54]

As already shown, 'the sum of prices of production for the commodities produced in society as a whole … is equal to the sum of their values.'[55] Yet, if the sum of prices had a double counting of profits the result would be wrong. Commodity inputs 'are generally bought on the market in capitalist production, so that their prices include an already realized profit … so that the profit in one branch of industry goes into the cost price of another.' When the sum of prices is considered, is this profit counted twice, once as the profit of the input producer and again as the profit contained in the cost price of the purchasing capitalist?

Let us assume that a linen producer requires only one input, flax. This commodity is purchased at its production price PP_f, which is, as in all production prices, made up by its cost price CP_f plus its profit P_f. The price of production of flax constitutes, therefore, the cost price of the linen. The prices of production of flax and linen can be written in the following form:

$$PP_f = CP_f + P_f$$
$$PP_l = CP_l + P_l$$
$$= PP_f + P_l$$
$$= (CP_f + P_f) + P_l$$

Since the profit of flax is an element of the cost price of linen, Marx wonders whether the sum of production prices of all branches would not contain the flax profit P_f twice, once in the price of flax (first line) and again in the price of linen (last line).

To answer this question Marx argues that, when considering 'the total social product', it is possible to 'put on one side the sum of the cost prices of all the commodities and the sum of the profits or surplus values on the other'. To do this, he suggests a procedure developed in *Capital* I, called the 'representation of the value of the product by corresponding proportional parts of the product'.[56] In this passage, he treats 'the product of any capital … as if one part simply replaces *capital*, while the other only represents *surplus value*.' Analogously, when a commodity is exchanged at its price of production, the latter can be broken down into *cost price* and *profit*. 'To apply this method of reckoning to the social product', Marx concludes that, by summing all the prices, the profit embodied in the inputs is added only once, that is that 'the profit contained in the price of flax, for instance, cannot figure twice, not as both part of the price of the linen and as the profit of the flax producers'. Therefore, for the whole society 'there is no distinction between profit and surplus value' and, thus, the equality between global prices and values is not affected.

Table 3.8 is a reworking of Table 3.7 – the final result of the iterative transformation procedure – according to the method suggested by Marx. As already seen, Table 3.7 encompasses all productive spheres; to carry out the exercise only the three global aggregates will be considered: department I is formed by the spheres producing means of production (spheres 3 and 4), department II by those producing wage goods (spheres 1 and 5) and III by the sole luxury goods producer (sphere 2). The construction of Table 3.8 will be illustrated for department I.

The price of production of every department has three components, c, v and p. Each of them can be, at the same time, broken down into 'one part that represents *cost price* while the other represents *profit*', as done with the price of production of linen. Considering this, the production price of department I, as all production prices, can be written as:

$$PP_I = CP_I + P_I$$
$$= PP_I(CP_I/PP_I + P_I/PP_I)$$

$$= PP_I(\alpha_1 + \text{ß}_1)$$

Table 3.7 shows that department I's cost price and profit are 150.9 and 41.7 respectively, so $\alpha_1 = 0.783$ and $\text{ß}_1 = 0.217$. Analogously, department II and III prices of production can be broken down into two fractions, one corresponding to cost price and another to profit, which implies the following proportions: $\alpha_2 = 0.680$, $\text{ß}_2 = 0.320$, $\alpha_3 = 0.796$ and $\text{ß}_3 = 0.203$. In this manner, department I's *constant capital* (as well the constant capital of other departments) can be separated into 78.3 per cent, corresponding to cost price, and 21.7 per cent representing profit, whereas its *variable capital* can be separated into 68 per cent for cost price and 32 per cent for profit. This calculation would not be complete unless the part of the production price corresponding to *profit* is similarly broken down. Since profits are used in the purchase of commodities produced by III, then they must be separated by using the proportions in which the price of production of III is divided into cost price and profit, that is 79.6 per cent and 20.3 per cent. Therefore, the three elements of department I's production price can be divided as follows:

$$192.6_I = 87.7_I^c + 63.2_I^v + 41.7_I^p$$
$$192.6_I = 87.7_I^c(\alpha_1 + \text{ß}_1) + 63.2_I^v(\alpha_2 + \text{ß}_2) + 41.7_I^p(\alpha_3 + \text{ß}_3)$$
$$192.6_I = \{(68.7 + 19.0)_I^c + (43.0 + 20.2)_I^v + (33.2 + 8.5)_I^p\}$$

(where subscripts indicate department and superscripts show the element of production price). If this calculation is worked out for all departments, table 3.8 is obtained.

In the above formulas, the sum of the first element of each set of parentheses (for department I: $68.7 + 43.0 + 33.2 = 144.9$) represents the *cost price* of the elements which form the production price of each department; the sum of the second element ($19.0 + 20.2 + 8.5 = 47.7$) represents the *profit* of the elements constituting this price of production. These figures, corresponding to each department, appear in the last two columns of Table 3.8. If both magnitudes for all departments are vertically summed, Marx's suggestion is followed: 'if the sum of cost prices of all commodities in a country is put on one side and the sum of the profits or surplus values on the other, we can see that the calculation comes out right.' Indeed, the vertical sum of the last column of Table 3.8 clearly indicates that the sum of the profits embodied in prices of production corresponds to the sum of profits or surplus values in Table 3.7. This shows that, taking the totality of commodities into account, the method for transforming values into prices of production *does not imply* a double counting of profits, which is the problem set forth by Marx in the above mentioned passage.

In most writings on the transformation problem, this passage has been interpreted in two different ways. First, it has been quoted as evidence that Marx was aware of the fact that input values must be transformed although, for some reason, he did not follow his example up to their final outcome. Yet, it can be thought that Marx implicitly assumes the calculation has been completed and that

he contrasts the *conceptually* achieved results – not those *arithmetically* unfinished in his *draft* – with the possibility of a double counting of profits.

	Constant Capital Used Up			Variable Capital			Profit			Production Price		
	S	Cost Price	Profit	S	Cost Price	Profit	S	Cost Price	Profit	S	Cost Price	Profit
I	87.7	68.7	19.0	63.2	43.0	20.2	41.7	33.2	8.5	192.6	144.9	47.7
II	57.2	44.8	12.4	28.7	19.5	9.2	40.5	32.2	8.2	126.4	96.6	29.8
III	47.7	37.3	10.3	34.5	23.4	11.0	20.9	16.7	4.3	103.1	77.5	25.6
Σ	192.6	150.9	41.7	126.4	85.9	40.5	103.1	82.1	20.9	422.0	318.9	103.1

Table 3.8

A second interpretation, as mentioned earlier, has been recently advanced by several authors and has become known as the New Solution to the transformation problem.[57] These writers recognize the problem presented by Marx in the above passage, although they do not understand the nature of his final answer. Since this interpretation now has significant consensus, it is necessary to consider the conclusions it draws from this passage. In particular, the following has been frequently mentioned:

> To apply this method of reckoning to the total social product, we have to make certain rectifications, since, considering the whole society, the profit contained in the price of flax, for instance, cannot figure twice, not as both part of the price of the linen and as the profit of the flax producers. (Marx 1981:260)

What are these *rectifications* that one should make? According to the New Solution authors, Marx suggests that, when all prices are considered, there is a double counting of the *constant capital* consumed; they deduce that 'the rectifications' consist of suppressing the elements of the constant capital from the sum of values and prices and only take into account the 'net product', that is the value product v + s.[58] As will be immediately seen, this interpretation contradicts the meaning and the conclusions of Marx's passage quoted above.

A few lines before the sentence just quoted, Marx calls k the cost price of all the inputs of a given commodity, p the profit embodied in them and p_1 the profit on the commodity itself. In department I of Table 3.8, Marx's calculation would appear as $k = 68.7 + 43.0 = 111.7$, $p = 19.0 + 20.2 = 39.2$ and $p_1 = 41.7$ (Table 3.7). If this department is considered alone 'the total profit [is] $P=p + p_1$', that is 80.9. However, if such a calculation were to be carried out for all departments, a double counting effect of the profits would indeed occur, since 'the profit contained in the price of flax, for instance, cannot figure twice, not as both part of the price of the linen and as the profit of the flax producers.' Thus, the formula $P = p + p_1$ must be *rectified* to calculate the global profit in one of two ways. Either, as already done in Table 3.8, each component of the price of production (c, v and p) can be broken down into cost price and profit; only then it is possible to sum the parts representing the profit of all departments (47.7 + 29.8 + 25.6 = 103.1). Alternatively, the profits appropriated by the capitalists of each department must be considered individually (41.7 + 40.5 + 20.9 = 103.1). Marx

thus asks if there is a double counting of *profits*, and not of the *consumed constant capital.*[59]

Before concluding, it is interesting to note that, with regard to the procedure illustrated in Table 3.8, Marx mentions explicitly only the elements of the cost price. To divide the elements of the cost price into cost price and profit is an operation with a concrete reference: the cost price is a part of the price of production of the commodity which is the realized form of other commodities – the components of constant capital and wage goods. It is easy, therefore, to represent the commodity's cost price as made up of cost price and profit. Yet, this does not occur with the profit element of the production price: when the commodity is individually considered, this component does not constitute the realized form of any commodity and cannot be *immediately* broken down into cost price and profit. This separation can only be carried out when the whole economy is taken into account; in this case it is clear that the profit must be realized in a series of commodities whose production price can be separated into cost price and profit. For this reason, when Marx deals with an individual commodity, he investigates the elements of cost price but, when he treats production as a whole – for instance, when he discusses the sum of prices and the sum of values – he considers all elements of the price of production, including those that correspond to profit.

Second Problem: when all commodities (including the *inputs*) are exchanged at production prices, is there a global cancellation of the divergences between values and prices of production?[60]

When Marx presents the results of the transformation process, he considers the social capital and shows that the divergences between surplus values and profits are cancelled out. Since the difference between values and prices is only the difference between surplus value and profit, this cancellation implies that the divergences between values and prices are cancelled out for the social capital – as shown in Table 3.7. In the text Marx wonders whether this result is maintained when the elements of cost price are contemplated:

> Apart from the fact that the price of the product of capital B, for example, diverges from its value, because the surplus-value realized in B is greater or less than the profit added in the price of the products of B, the same situation also holds for the commodities that form the constant part of capital B, and indirectly, also, its variable capital, as means of subsistence for the workers. (Marx 1981:261)

As in the text commented on above, Marx faces the fact that all commodities, including those consumed as inputs, are exchanged at prices diverging from their values. Accordingly, the value crystallized in the elements of the cost price diverges from the respective price of production. The problem raised by Marx is whether these divergences are cancelled out in the economy as a whole, thus causing the total sum of production prices to be identical to the sum of values. When the linen producers purchase flax in order to consume it in their production

processes, they pay for the flax at its price of production which generally does not coincide with its value. Is this divergence between the price of production of the flax and its value offset by other divergences with the opposite sign or, instead, is it added to other divergences? In other words, if the prices of production of all inputs are put on one side, and the values on the other, are all divergences reciprocally cancelled out?

Marx answers affirmatively: 'whenever too much surplus value goes into one commodity, too little goes into another'. Citing the example in Table 3.7, Table 3.9 applies this reasoning to all departments in the economy. As in Table 3.8, the components of the production price (c, v and p) of the various spheres of production are broken down into two parts; but, in this case, they are divided into the *value* and the *divergence* between price of production and value. Let us examine an example to see how Table 3.9 is set up.

The price of production of I – or any other department – can be written as:

$$PP_I = VA_I + (PP_I - VA_I)$$
$$= PP_I\{VA_I/PP_I + (PP_I - VA_I)/PP_I\}$$
$$= PP_I(\delta_1 + \gamma_1)$$

In Table 3.7, I's price of production is 192.6 and its value is 202.4, so that $\delta_1 = 1.051$ and $\gamma_1 = -0.051$. With these coefficients, the constant capital consumed by all departments can be broken down into two parts: 105.1 per cent corresponding to the *value* of the means of production used up and –5.1 per cent corresponding to the *divergence* between their price of production and value. The same calculation can be worked out for production prices of the other departments, obtaining $\delta_2 = 0.865$, $\gamma_2 = 0.135$; $\delta_3 = 1.069$ and $\gamma_3 = -0.069$. The division of all the elements of I's price of production into one part corresponding to their value and the other corresponding to the divergence between value and production price has the following form:

$$192.6_I = 87.7_I^c + 63.2_I^v + 41.7_I^p$$
$$192.6_I = 87.7_I^c(\delta_1 + \gamma_1) + 63.2_I^v(\delta_2 + \gamma_2) + 41.7_I^p(\delta_3 + \gamma_3)$$
$$192.6_I = \{(92.2 - 4.5)_I^c + (54.7 + 8.5)_I^v + (44.6 - 2.9)_I^p\}$$

It is obvious that if only one *particular* department or sphere of production is taken into account, the sum of the divergences embodied in the elements of its cost price (–4.5 + 8.5), as well as those contained in the elements of its production price (–4.5 + 8.5 – 2.9) would not be nil.[61] The problem posed by Marx is to consider *all* the departments or spheres of production to find out whether or not the sum of the divergences is nil. Consequently, the calculation only makes sense if all departments or spheres of production are taken into account and, thus, if it is worked out for the *global* sum of divergences. This is the goal of Table 3.9.

It has to be emphasized once again that this procedure must be applied to all the elements of the production price, and not only to elements of the cost price, that is it must be also applied to commodities produced by III, where the capitalists' profit is realized. In the last two columns of Table 3.9, it can be seen

how the production price of each department is divided into one part which corresponds to the sum of the *values* embodied in its elements and into another corresponding to the sum of the *divergences* between the prices of these elements and their values. If all these accumulated divergences are added vertically, it is clear that 'the divergences from value that [are contained] in the production prices of commodities therefore cancel each other out.'[62] In this form, the theoretical result that the sum of prices corresponds to the sum of values is maintained.

	Constant Capital Used Up			Variable Capital			Profit			Production Price		
	Σ	Value	Diver-gence	Σ	Value	Diver-gence	Σ	Value	Diver-gence	Σ	Value	Diver-gence
I	87.7	92.2	-4.5	63.2	54.7	8.5	41.7	44.6	-2.9	192.6	191.4	1.1
II	57.2	60.1	-2.9	28.7	24.9	3.9	40.5	43.3	-2.8	126.4	128.2	-1.9
III	47.7	50.1	-2.4	34.5	29.8	4.6	20.9	22.4	-1.5	103.1	102.3	0.7
Σ	192.6	202.4	-9.9	126.4	109.3	17.0	103.1	110.2	-7.2	422.0	422.0	0.0

Table 3.9

Third Problem: what would happen if the cost price of a commodity is equated to the *value* of its material elements?[63]

In the third passage to be discussed here, Marx once again recalls that the cost price of the commodities coincides with the price of production – but not with the value – of their material elements:

> It was originally assumed that the cost price of a commodity equalled the value of the commodities consumed in its production. But for the buyer of a commodity, it is the price of production that constitutes its cost price and can thus enter into forming the price of another commodity. As the price of production of a commodity can diverge from its value, so the cost price of a commodity, in which the price of production of another commodities is involved, can also stand above or below … the value of the means of production going into it. (Marx 1981:264-265)

It is clear that during a previous analytical phase, when commodities were exchanged at their values, the cost price *was* equal to the *value* of its material elements. Once this assumption is dropped, the cost price must be equated to the price of production of its material components. In the new analytical step, the cost price has, thus, a '*modified significance*' with regard to its *original* situation.

The specific problem posed by Marx in this third passage is the following: what *would happen if*, once the transformation is accomplished, the cost price is equated to the *value* of its material components, that is to the *value* of the means of production and the *value* of the wage goods consumed in its production, rather than with their *price*? Marx's answer to this hypothetical problem is clear: there would be an error ('it is always possible to go wrong') because 'this modified significance of the cost price' would have been disregarded:

> It is necessary to bear in mind this modified significance of the cost price, and therefore to bear in mind too that if the cost price of a commodity is equated with the value of the

means of production used up in producing it, it is always possible to go wrong. (Marx 1981:265)

Marx's numerical calculations in his third table have been left unfinished and, since in this table the cost price is equated to the value rather than the price of the means of production, it is possible 'to go wrong'. But, if the cost price is correctly considered, the possibility of such an error disappears.

On the basis of the calculations in Table 3.9, it is possible to see what happens if this 'modified significance of the cost price' is disregarded. Let us suppose that the *value* and not the *price* of the material elements used up is considered in the calculation of the cost prices. In this case, I's cost price would be 92.2 + 54.7 = 146.9; II's cost price would be 60.1 + 24.9 = 85.0, and III's would be 50.1 + 29.8 = 79.9. If the production prices of each branch were not calculated considering the 'modified significance' of their cost prices (150.9, 85.9 and 82.1, as in Table 3.7), but the profits of each department were added to 'cost prices not transformed', a different calculation of production prices would be attained (namely, 146.9 + 41.7 = 188.6; 85.0 + 40.5 = 125.5 and 79.9 + 20.9 = 100.8) whose sum (414.9) would not correspond to that of the values, 422. This leads to a conclusion that the sum of prices is equal to the sum of values only if the 'modified significance' of the cost price is considered and, thus, in the calculation of production prices the inputs 'are transformed'.

3.8   CONCLUSIONS

In the numerical examples which Marx drafted to illustrate the transformation of values into prices of production, the sum of surplus values equals the sum of profits and the sum of values equals the sum of prices. In the wake of Tugan Baranowsky and Bortkiewicz, virtually all other authors writing on the transformation agree that this result only arises in Marx's tables because 'the transformation has not been concluded': the inputs are exchanged at their values and not at their production prices. There have been several dualistic attempts 'to correct the transformation', either through simultaneous equation systems – such as Bortkiewicz's (1984), Winternitz's (1948) and Seton's (1957) – or through iterative approaches – such as Shibata's (1933), Bródy's (1970) and Shaikh's (1977). All these attempts have reached the same conclusion: insofar as Marx 'is corrected', his double equality is invalid.

Since this conclusion has important implications for Marx's entire theoretical framework, the subsequent debate has involved many authors and has been the longest ever in the history of economic thought. However, all these attempts to 'correct' the transformation are grounded on an understanding of value which differs from Marx's. For the usual approach, value is a magnitude determined once and for all in the sphere of production which is related to price in a purely exterior manner. Marx's transformation problem has been reduced into a Ricardian problem: to find a direct relation between labour (or 'the sphere of

value') and prices of production (or 'the sphere of prices'). If, on the contrary, the relation between value and its form is considered in dialectical terms, the value and price of the commodities are determined both qualitatively and quantitatively as the result of the same process, namely capitalist competition. From this point of view, the transformation problem is no longer formulated in Ricardian terms.

The method used in this chapter to rework Marx's numerical example has sought to express the relation between values and prices. Once the solution is reached, all inputs and outputs are exchanged at their prices of production and, at the same time, the double equality enunciated by Marx holds. After Marx conceptually (though not numerically) concluded the transformation, he tested it in a group of passages which have been repeatedly quoted, often out of context. A thorough reading of these texts has shown that Marx proved his methodology to be right. In this chapter, a numerical illustration of Marx's test of his solution has shown the soundness of his theoretical proposal and conclusions.

ACKNOWLEDGEMENTS

NOTES

[1]   On the previous debate, see Bortkiewicz (1952), Dostaler (1978) and Howard and King (1989). Mühlpfort was a predecessor of Bortkiewicz, recently rediscovered by Howard and King (1989).

[2]   Bortkiewicz (1952: 8).

[3]   As Kliman and McGlone point out in this volume, 'the non-dialectical understanding perceives each object as isolated, uniquely itself, a whole unto itself.'

[4]   Nomenclatures differ. In this chapter the authors refer to the 'New Approach' as the 'New Solution' – editors.

[5]   Recent surveys of the literature (Desai 1989 and Bellofiore 1989) do not contain any available contribution from writers of the anti-dualistic vision.

[6]   Marx (1976a: 421). Oakley (1976) shows that Marx had already delineated the solution of this contradiction in *Grundrisse* (Marx 1973:435-6).

[7]   For example, Hunt and Glick (1987:356): 'the value of a commodity consisted of the labour embodied in the means of production … (dead labour) and the labour expended in the current production period (living labour).'

[8]   'although exchange value is = to the relative labour time materialized in products, money, for its part is = to the exchange value of commodities, separated from their substance'. (Marx 1973:160)

[9]   Marx (1976b:34). This text is from the first German edition of Chapter 1 of Volume I of *Capital*.

[10]   See notes 6, 11, 18, 26, 29, 53, also Marx to Engels 16 January 1858 and 30 April 1868, Marx to Kugelman 4 July 1868, Marx 1969b:25-30, 173-175, 190-191, 198-199; 1972:81-84, 87-88, 164-165, 167-168, 176-177; 1991:36-7, 75-6, 83-4, 94-101, 232, 1972:330-333, 376-377.

[11]   The first tabular solution (2 tables, 5 spheres) is presented in *Theories of Surplus Value* II (1969b:64-68); the second (1 table, 4 spheres) in a letter to Engels dated 2 August 1862; the third (1 table, 4 spheres) in *Theories of Surplus Value* II (1969b:389); the fourth (3 tables, 5 spheres) in *Capital* III, Chapter 9 (1981:254-9) and the fifth (1 table, 3 spheres) in the same chapter (p. 263-4). If literature

were concerned with this last solution, the creation of a 'system of values' entirely separated from prices would be more difficult.

[12] Because we do not consider the matter without fixed capital, Marx's first table is not commented on.

[13] Marx (1981:256). The only changes introduced in the table are the order of the columns and the suppression of the column corresponding to the rate of surplus-value.

[14] Tugan Baranowsky with his 'labour-value' schema (1905:170-174) is the practically unknown founder of this approach.

[15] Marx (1976a:188). In his critique of labour-money Marx asks: 'Since labour-time is the intrinsic measure of value, why use another extraneous standard as well? Why is the value … computed in terms of an exclusive commodity, which thus becomes the adequate expression of exchange value, i.e. money?' (Marx 1970:84). See Saad-Filho (1993) for a discussion of Marx's critique of labour-money. Rubin (1973:111-113) warned against the usual practice of measuring value in labour-time instead of measuring it, as Marx does, in money. Elson (1979b:135-139) provides a lucid discussion about this common error.

[16] See notes 6 and 11.

[17] Marx (1969b:67); first emphasis added. Also: 'Let us assume … that all commodities … were sold at their actual values … i.e. [that] they are exchanged with one another in proportion to the value contained in them, at their *value prices*' Marx (1981:275, emphasis added).

[18] Marx says that throughout *Capital* I and II he has 'in fact assumed that prices = values. We shall, however, see in Volume III that even in the case of average prices, the assumption cannot be made in this simple manner'. (Marx 1976a:329n). This assumption, explicitly made in *Capital* I, Chapter 5 (1976a:268-9) and maintained in the first two tables of *Capital* III, Chapter 9, does not imply that the figures of these tables are magnitudes of labour. Rather, it means that the exchange relations between commodities (their prices) are directly proportional to the amounts of labour congealed in them.

[19] 'The rates of profit prevailing in the different branches of production are accordingly originally very different. These different rates of profit are balanced out by competition to give a general rate of profit which is the average of all these different rates. The profit that falls to a capital of given size according to this general rate of profit, whatever its organic composition might be, we call the average profit. That price of a commodity which is equal to its cost price, plus the part of the annual average profit on the capital applied in its production … is its price of production.' (Marx 1981:257-8).

[20] Marx (1972:82). In this passage, Marx still uses 'cost price' instead of 'price of production'

[21] Marx (1981:259, 273).

[22] For instance, Bortkiewicz (1952:16); Garegnani (1959:24, 211). Bortkiewicz quotes Marx's following passages: Marx (1978: 460, 464-5, 466, 478; 1981:985). See also Marx (1978:459; 1981:277).

[23] In general, the relation between price and value has to be interpreted in a Hegelian way: 'the essence is being that is past, but timelessly past' (quoted by Inwood 1992:90). In particular, when Marx criticizes Smith's conception, he refers to value as 'coming first' with respect to the components of individual commodities. Value is understood as the social regulating magnitude which has asserted itself upon individual capital as 'the law of gravity asserts itself when a person's house collapses on top of him.' Marx (1976a:168). It is on the basis of this social determination (that is, value) that – directly, in *Capital* I and II, or indirectly, in *Capital* III – the components of individual commodities are determined.

[24] Criticizing Rosdolsky's approach, Elson (1979b:168) says: 'The trouble with this explanation is that it often leads to confusion about competition: to the view, for instance, that *Capital* I, abstracts from competition. This is clearly not the case: competition is an essential feature of capitalism; capital can only exist in the form of many capitals.'

[25] Marx (1969b:174); emphasis omitted.

[26] Letter to Engels, 27 June 1867, emphasis omitted.

[27] In the *Manuscript 1861-63*, Marx considers this to be a 'first transformation' that – it can be interpreted – is represented in the first/second table. See Marx (1991: 96-101).

[28] 'At a given rate of surplus-value it is only for capitals of the same organic composition – assuming equal turnover times – that the law holds … that profits stand in direct proportion to the amount of capital … [a result that] is true on the same basis as our whole investigation so far: that commodities are sold at their values' (Marx 1981: 252). See also Marx (1991:299-300).

[29] 'This … conclusion only raises the question how on the basis of exchange-value a market-price differing from this exchange-value comes into being, or rather, how the law of exchange-value asserts itself only in its antithesis. This problem is solved in the theory of competition'. (Marx 1970:62).

[30] Marx (1981:274).

[31] Marx's equalities are not external postulates as generally and erroneously interpreted. Nor can a unity of opposites be reduced to a tautology as Böhm-Bawerk (1984:36) and Duménil do (1983:446) because it does not consist in defining a concept in terms of itself ($A \equiv A$). See Marx (1972:87-88).

[32] Using Platonic terminology, Desai has called these 'two worlds' 'the invisible value domain' and 'the visible price or exchange domain' (1979:143).

[33] Tugan Baranowsky (1905:174-175). See comments by Samuelson (1971) and Dostaler (1978) on Tugan's formulation.

[34] Bortkiewicz (1952) and, particularly, Bortkiewicz (1984).

[35] Bortkiewicz (1952:8).

[36] The last two columns and column 6, originally suppressed by Bortkiewicz, have been added to the table. The method used to calculate them and their function will be explained in the following section.

[37] Komorzynsky (1897:294, 289); our translation. The cited passage from Marx is 1981:779. Komorzynsky was Professor in the University of Vienna; Böhm-Bawerk edited the journal which published his article.

[38] Marx (1981:264-265). This passage is commented on in the present article (see the last section: 'Third problem').

[39] Bortkiewicz (1906:15 (German edition)); our translation.

[40] Tugan Baranowsky (1905:174); our translation. This argument will be noisily repeated by Steedman (1977) though inspired by Garegnani (1959).

[41] Bortkiewicz (1984:12) and Bortkiewicz (1984:205).

[42] Tugan Baranowsky (1905:174); our translation.

[43] Actually, Bortkiewicz's *dualistic* conception comes from Tugan Baranowsky's book where he tries to solve what was later called 'the inverse transformation problem' that is, how 'to transform prices of production into labour-values'. Here, Tugan Baranowsky shows an understanding of the relationship between value and price as a completely outward link: for instance, he thinks that the 'value schema' is expressed in 'labour-units', while the 'price-schema' is calculated in 'money-units'.

[44] For a justification of this approach – in particular for the thesis according to which the *value* transferred by the means of production to the final commodity does not correspond to their value but to their price – see Wolff, Roberts and Callari (1982, 1984a), Carchedi (1984), Roberts (1987), Kliman and McGlone (1988), Ramos (1991), Moseley (1993a) and Rodríguez (1994). Moseley (p. 171) mentions Mage (1963, Appendix A) and Mattick Jr. (1981) as supporters of this conception.

[45] Marx (1978:470-471).

[46] We have rounded off the errors in all the following tables.

[47] The iterative method has been used by Shibata (1933), Bródy (1970), Morishima (1973), Okishio (1974) and Shaikh (1977). Using the categories of individual and social value, Carchedi (1984) develops a temporal determination of values and prices that goes beyond the mere calculation process.

[48] One example of a simultaneous equation system was presented in Rodríguez (1994). The conditions of equivalence between the iterative and the simultaneous equations solutions were presented in Laise, Pala and Valentino (1977) and Panizza (1981). See also Giussani (1991), Freeman (1993a), Kliman (1993) and Naples (1993). The use of 'postulates of invariance' is only possible (and necessary) in the context of the dualistic approach. These 'postulates' are the conjunction of two conditions of normalization, one in the 'system of values' and another in the 'system of prices'. The relationship between both normalization conditions makes it possible to define a different unit of measure in each 'system' in such a way that one aggregate equals any figure in the other. The above-presented iterative sequence does not consist of two different 'systems' but, rather, of a single system. Its solution gives both the vector of values and the vector of prices and needs only one condition of normalization. It is clear that Marx's equalities are verified for every iteration, independently of the selected normalization. Moreover, if in Table 3.7 use-value #1 is chosen as money-commodity, all the data have to be divided by 3.04 in order to make its 'price' equal to 1. This procedure changes the original normalization and,

therefore, the corresponding total value and total price become 138.9, different from the initial total of 422.0 (See Rodríguez's contribution in this volume).

[49] See Rodríguez's contribution in this volume, Ramos (1994) and Rodríguez (1994) for a discussion about this ratio. See also Aglietta (1979), De Vroey (1981) and Foley (1982).

[50] As his predecessor did 70 years earlier, Steedman (1977:29-31) imagines that there are two different rates of profit. From his misunderstanding he deduces his 'criticism is sound and cannot be answered.'

[51] Marx (1981:273).

[52] Bortkiewicz (1952:28).

[53] The reference for each text is presented after each question where the problem is summarized. These passages deal with the generically-named 'transformation of inputs problem'. Marx first considers this in two passages of the *Manuscript 1861-63* (see Marx 1972:166-168 and 1991:36-7); in *Capital* III he studies this generic problem in four diverse aspects, three of which are commented on here. The remaining aspect regards the average composition commodity and is less controversial (see Marx 1981:309). The first passage quoted (Marx 1972) plays an important role in the non-dualistic interpretation of Wolff, Roberts and Callari (1982:575).

[54] Marx (1981:259-261); from 'This seems contradicted … ' to ' … cannot figure twice'.

[55] Marx (1981:259).

[56] The method is presented in *Capital* I, Chapter 9 Part 2, (1976a:329-332), where Marx says that it will be applied later 'to complex and hitherto unsolved problems' (p. 331); the reader can also refer to a similar procedure developed in *Theories of Surplus Value* I, Chapter 3, Section 10 (1969a, pp. 107-151) and in *Capital* II, Chapter 3 (1978:169-171). As far as we know, this observation, included in one of the key texts of the transformation problem, has been neglected by the literature. It is worthwhile to note, however, that Schmidt (1971) uses the method suggested by Marx.

[57] Duménil (1980, 1983), Foley (1982), Lipietz (1982).

[58] Duménil (1980:37-39, 62-65); Glick and Ehrbar (1987:297-299). Marx calls V + S *Wertprodukt* (value-product). See Marx (1976a:669).

[59] According to Duménil, who quotes only two small parts of the above-mentioned text: 'As it is presented in *Capital* III this calculation [the rectifications] is quite incomprehensible' (1980:63); our translation. Yet, a *complete* and *careful* reading of the text reveals that this calculation is quite comprehensible. In particular, tracing Marx's suggestion regarding the possibility of dividing the elements of production prices into their component parts gives the text a clear meaning. However, the ensuing conclusion does not show that it is necessary to consider only the 'net product' to transform values into production prices, as maintained by Duménil, Lipietz and Foley. Glick and Ehrbar (1987:299) likewise slash up the text arbitrarily because Marx does not speak of a double counting 'in the reduction of profits' – as they call the difference between price and value in the flax industry – in relation to a hypothetical 'system of values'.

[60] Marx (1981:261); from 'The distinction is …' to '… the dominant tendency'.

[61] These divergences are nil only in the production price of an average-composition commodity.

[62] Marx (1981:261). (In the Penguin translation, 'obtain' replaces the words between brackets; the German original says '*stecken*'.)

[63] Marx (1981:264-265); from 'The development given … ' to '… on this point'.